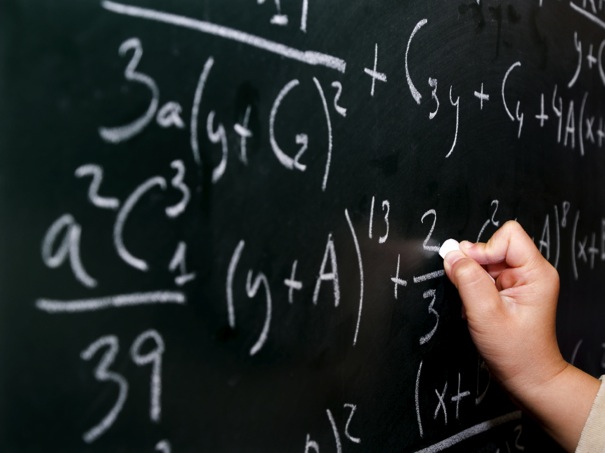
**REPUBLIQUEALGERIENNE DEMOCRATIQUE ETPOPULAIRE**

**MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH**

**UNIVERSITY Dr YAHYA Farès DE MEDEA**

COLLECTION DE LA FACULTE DE TECHNOLOGIE

**Physics course1: Mechanics of the material point**

**Intended for the first year Common Core ST**

**Title:**

Physique 1 **Mécanique du point matériels**

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**Year: 2022/2023**

**Approved by the Scientific Council of the Faculty on :**

**Table of Contents**

**General Introduction .............05**

**Chapter 1: Mathematical Reminders**

**1- Equations with dimensions.............06**

**2- Vector calculus ................ 09**

**Scalar product (norm), ...................... 11**

**Cross product........................................................................................ 12**

**3- Vector analysis ................ 12**

**4- Operators ................ 14**

**Gradient..................................................................................................... 14**

**Curl................................................................................................ 15**

**Discrepancy............15**

**Laplacian................................................................................................ 16**

**Chapter 2. Kinematics**

1. **Position vector in coordinate systems ................................................. 19**

**-Cartesian........................................................................................... 19**

**-Cylindrical. ....................................................................................... 20**

**-Spherical ............**

**-Curvilinear................................................................................................. 21**

**- Law of motion ................ 22**

**-Trajectory................................................................................................. 22**

**2- Speed and acceleration in coordinate systems. ........................................23**

**3- Applications: Movement of the material point in the different coordinate systems .28**

**4- Relative movement. ........................................................................................37**

**Chapter 3. Dynamic**

1. **General ............42**

**Mass - Force - Moment of force - Absolute and Galilean reference frame ........................ 42**

1. **Newton's laws. .................................................................................... 46**
2. **Principle of conservation of momentum.................................... 46**
3. **Differential equation of motion. .......................................................... 50**
4. **Angular momentum. ..................................................................................... 57**

**6- Applications of the fundamental law for forces (constant, time dependent, velocity dependent, central force, etc.). ......................................................... 58**

**Chapter 4 Labour and Energy**

1. **Work of a force. .................................................................................... 61**
2. **Kinetic energy. ...................................................................................... 63**
3. **Forces conservatives et non conservatives …………………………………………........ 64**
4. **Potential energy ................ 66**

**Examples of potential energy (gravity, gravitational, elastic). .................66**

1. **Total energy theorem. ........................................................................ 67**

**References........................................................................... 70**

**The objective of the course**

**The objective of the course is to introduce students to the fundamentals of Newtonian physics through three main sections: Kinematics, Dynamics, and Work and Energy.**

**General introduction**

This handout presents courses on the mechanics of the material point and some exercises. It is intended for first-year ST. It is a question of studying the movement of material bodies as a function of time (kinematics), and studying the forces that cause or modify their movement (dynamics).

This handout is subdivided as follows:

**The first chapter** is devoted to a mathematical reminder on the dimensional and vector analysis that are necessary to express the laws of physics.

We determine the notion of dimension and the equations to dimensions then we recall the notion of the vector, then we present the operations on the vectors: the sum, the subtraction and the product of the vectors and we finish this part with the differential operators (nabla operator, gradient, divergent, rotational and the Laplacian).

**The second chapter** is intended for the kinematics of the material point. We present the descriptive study of the motion of a point by determining the position, displacement, velocity and acceleration vector in the different coordinate systems. Next, we study the different types of movement. We conclude this part with the study of relative motion. It is a question of studying the movement of a point with respect to a moving reference frame (moving reference frame). We discuss the case of a coordinate system in translational motion and the case of a coordinate system in rotational motion.

**The third chapter** of this handout is devoted to the dynamics of the material point. We introduce the notion of forces, mass and the principle of inertia. We then present Newton's three laws of dynamics and study the different forces (contact forces, frictional forces, elastic forces and inertial forces). We conclude this chapter with the determination of angular momentum and central forces.

Finally**, the fourth chapter** is dedicated to work and energy where we have defined the notions of work, kinetic energy and potential energy. Then we turned our attention to conservative and non-conservative forces. We end this chapter with the notion of the total or mechanical energy of a system.

**Chapter 1: Mathematical Reminders**

**1.1-Dimensional Analysis**

**1.1.1 Definitions**

Dimensional analysis is the study of the general form of physical equations. It makes it possible to quickly check the consistency of the physical quantities used to write these equations.

**1.1.2. Dimension of a physical quantity**

The dimension is a property associated with a unit. It is denoted **[G]** and it is called dimension of the magnitude G.

It is very general, and it does not depend on systems of units.

It seems like you’re referring to the concept of dimensional analysis in physics. In dimensional analysis, physical quantities are expressed in terms of fundamental dimensions. The seven fundamental dimensions typically used in this analysis are showed in the following table

|  |  |  |  |
| --- | --- | --- | --- |
| Quantities | Symbols | Units in (SI or MKSA) | Symbols |
| Length |  |  |  |
| Mass |  |  |  |
| Time |  |  |  |
| Intensity of electric current |  |  |  |
| Temperature |  |  |  |
| Intensity of the luminous current |  |  |  |
| Amount of material |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Quantities | Symbols | Units in (SI or MKSA) | Symbols |
| Length | L | Meter | m |
| Mass | M | Kilogram | Kg |
| Time | T | Second | s |
| Intensity of electric current | I | Ampere | A |
| Temperature |  | Kelvin | K |
| Intensity of the luminous current | J | Candela | cd |
| Amount of material | N | Mole | Mol |

All other quantities are related to these fundamental quantities.

**1.1.3. Dimensional equations**

These are mathematical relations giving the dimension of the quantity (using its symbol), which make it possible to verify the homogeneity of the formulas. All physical quantities can be expressed on the basis of these seven fundamental dimensions. So, a physical quantity G can be broken down according to the relation:

With K a dimensionless constant ([K] =1)

These fundamental dimensions are used to derive

These fundamental dimensions are used to derive units for all other physical quantities. For example, velocity can be expressed as [L/T], acceleration as [L/T^2], force as [M\*L/T^2], and so on. The idea is that any physical equation must have dimensions that balance on both sides to ensure dimensional homogeneity.

On both sides of the equation, the dimensions match, indicating that the equation is dimensionally consistent.

Dimensional analysis is a useful tool for checking the validity of equations, deriving relationships between physical quantities, and ensuring that the units are consistent in a given formula or equation.

Dimensional analysis is a mathematical technique used in science and engineering to check the consistency of equations and to convert between different units of measurement. It is a powerful tool that relies on the principle that physical quantities can be expressed in terms of fundamental dimensions, such as length, mass, time, and electric charge. By analyzing the dimensions of different quantities and ensuring they are consistent, dimensional analysis can help identify errors in equations and provide insight into the relationships between physical quantities.

Here are the key steps involved in dimensional analysis:

1. Identify the Fundamental Dimensions: Determine the fundamental dimensions relevant to the problem. Common fundamental dimensions include:
   * Length (L)
   * Mass (M)
   * Time (T)
   * Electric Charge (Q)
2. Express Physical Quantities in Terms of Fundamental Dimensions: Write down the dimensions of each physical quantity involved in the problem. For example, if you have a velocity (v), you would express it as [L][T]⁻¹, indicating that it has dimensions of length per time.
3. Use Dimensional Homogeneity: For equations to be physically meaningful and consistent, they must have the same dimensions on both sides of the equation. This is known as dimensional homogeneity. Check that all terms in your equations have consistent dimensions.
4. Apply Dimensional Analysis for Unit Conversion: You can also use dimensional analysis to convert between different units of measurement. For example, if you want to convert a length from meters (m) to feet (ft), you can use the fact that 1 meter is approximately equal to 3.281 feet to set up a dimensional equation and solve for the conversion factor.
5. Solve for Unknowns: In some cases, you can use dimensional analysis to derive relationships between physical quantities or to solve for unknowns in equations.

Dimensional analysis is particularly useful in physics and engineering to verify equations, design experiments, and perform unit conversions. It's a powerful tool for maintaining the integrity of mathematical models in scientific research and practical applications.

Top of Form

**Example 1**

Complete the following table with dimensional equations

|  |  |  |
| --- | --- | --- |
| Quantities | Symbols | Equations with dimensions |
| Angle |  |  |
| Acceleration |  |  |
| Density |  |  |
| Power |  |  |
| Frequency |  |  |
| Strength |  |  |
| Energy |  |  |
| Electrical voltage |  |  |
| Magnetic field |  |  |
| Electric charge |  |  |
| Electric field |  |  |

|  |  |  |
| --- | --- | --- |
| Quantities | Symbols | Equations with dimensions |
| Angle |  |  |
| Acceleration |  |  |
| Density |  |  |
| Power |  |  |
| Frequency |  |  |
| Strength |  |  |
| Energy |  |  |
| Electrical voltage |  |  |
| Magnetic field |  |  |
| Electric charge |  |  |
| Electric field |  |  |

**Example 2**

Find the dimensions of the functions f (r, t), g (r, t) and h (r, t) of the following homogeneous equation:

**Solution**

To find the dimensions of the functions f(r, t), g(r, t), and h(r, t) in the given homogeneous equation, we can use dimensional analysis. In dimensional analysis, we assign dimensions to the variables and constants and then determine the dimensions of the functions based on the units of other quantities in the equation.

For the equation to be homogeneous, all terms on both sides of the equation must have the same dimensions. These dimensions ensure that the equation remains homogeneous, meaning that all terms have the same dimensions on both sides. Therefore, we must have:

Top of Form

,

and

Therefore



**Example 3: Period of a simple pendulum**

The period P of a simple pendulum could depend on the **length l** of the wire, the **mass m** of the body and the **acceleration of gravity g**. Establish the relationship that describes this dependency

**Solution**

Expression of *P* as a function of other quantities

By comparison we will have

What gives

This formula is derived from the principles of simple harmonic motion and gravitational acceleration. It tells us that the period of a simple pendulum is directly proportional to the square root of the length of the pendulum wire and inversely proportional to the square root of the acceleration due to gravity. This means that a longer pendulum will have a longer period, and a stronger gravitational field will also result in a longer period.The mass of the bob (m) does not appear in this formula, which means that the period of a simple pendulum is independent of the mass of the bob.

**1.1.4. MKSA and CGS Unit Systems**

The MKSA (meter-kilogram-second-ampere) and CGS (centimeter-gram-second) unit systems are two different systems of units used in physics and engineering to measure various physical quantities. These systems are based on different sets of fundamental units and have different scales for many common physical quantities.

1. MKSA System:
   * MKSA is sometimes referred to as the "rationalized MKS system" or the "International System of Units" (SI), which is the modern metric system used worldwide for scientific and engineering purposes.
   * In the MKSA system, there are four fundamental units:
     + Meter (m) for length.
     + Kilogram (kg) for mass.
     + Second (s) for time.
     + Ampere (A) for electric current.
   * Other units are derived from these fundamental units using mathematical relationships. For example, the unit of force in MKSA is the newton (N), which is derived as 1 N = 1 kg·m/s².
   * The MKSA system is widely used in scientific research, engineering, and everyday measurements.
2. CGS System:
   * The CGS system is an older system of units and was widely used in the past, particularly in the late 19th and early 20th centuries. It has largely been replaced by the MKSA/SI system in most scientific and engineering applications but is still used in some specialized fields.
   * In the CGS system, there are three fundamental units:
     + Centimeter (cm) for length.
     + Gram (g) for mass.
     + Second (s) for time.
   * Other units are derived from these fundamental units. For example, the unit of force in CGS is the dyne (dyn), which is derived as 1 dyn = 1 g·cm/s². and 1 dyne is equal to 0.00001 newtons in the SI system

It's important to note that while the MKSA system is the more commonly used and internationally accepted system today, the CGS system is still encountered in some specialized fields, such as certain branches of astrophysics and plasma physics. However, in most modern scientific and engineering contexts, the SI system is the standard.

There are other systems of units such as English units (inch, foot, inch, thousand.) 1inch = 1/36 foot

**Example 1**

Complete Table 1 with the units of each quantity.

**Example 2**

Find units of functions f (r, t), g (r, t) and h (r, t)

**Solution**

The unit of f(r, t) is kg m S-2

The unit of g(r, t) is kg m S-2

The unit of h(r, t) is kg m 2 s-2

**1.2. Vector analysis**

**1.2.1. Scalar and Vector Quantities**

In physics, two types of quantities are used: scalar quantities and vector quantities.

**Scalar physical quantity** is entirely defined by a number and an appropriate unit.

**Vector physical quantity** is a quantity specified by a number, an appropriate unit, a path and direction**.**

**Example**

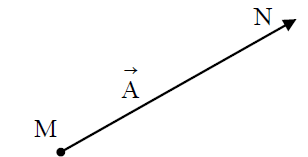
The mass is a scalar quantity.

Velocity is a vector quantity.

Vector analysis, also known as vector calculus, is a branch of mathematics that deals with vector fields and the differentiation and integration of vector functions. It is an essential tool in various fields of science and engineering, particularly in physics and engineering disciplines. Vector analysis is used to describe and analyze physical quantities that have both magnitude and direction, such as force, velocity, and electric and magnetic fields.

So, vector analysis is a powerful mathematical tool that is used extensively in physics, engineering, and other fields to model and solve problems involving physical quantities and their interactions. It plays a crucial role in understanding and describing the behavior of various systems.

**1.2.2. Vector**



A vector is an oriented segment that has:

- an origin M;

- a magnitude: the length of the segment.

-one direction: (MN)

- one path: from M to N.

We can say that the vector is a quantity that have both magnitude and direction. It can be represented geometrically as arrows in space, with the length of the arrow representing the magnitude of the vector, and the direction indicating its direction.

**Comment**

A vector can be designated by a single letter, for example:

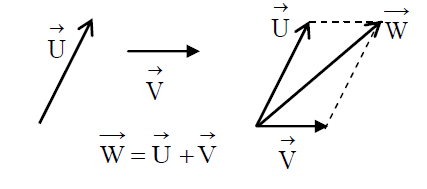
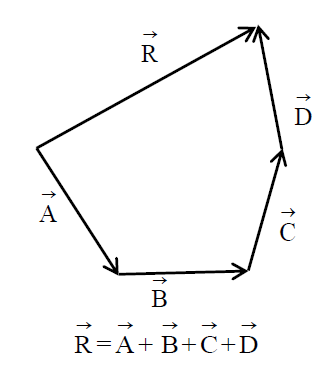
**1.2.2.1. Elemental operations on vectors**

**-Vector addition**

Vectors can be added together using the parallelogram law or the triangle law. The result is another vector.

The **sum** of two free vectors , denoted , is a free vector , obtained by the "parallelogram law"

When the number of vectors to be added is greater than two, the geometric method of placing them end to end is applied as shown in the following figure.

** **

**Properties**

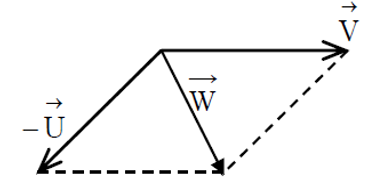
Commutativity

Distributivity with respect to vectors

**-Vector subtraction**

Subtracting one vector from another is done by adding the negative of the second vector.

Given two vectors , the difference is obtained with the parallelogram rule (see next figure)



**Chasles’s relation**

Chasles's relation, also known as Chasles' theorem, is a fundamental principle in vector mathematics and physics. It states that the sum of a series of vectors that form a closed polygon (a polygon where the initial and final points coincide) is zero. In other words, if you start and end at the same point when adding a series of vectors together, their sum will be a zero vector. This relation is named after the French mathematician Michel Chasles, who made significant contributions to the study of geometry and vectors in the 19th century.

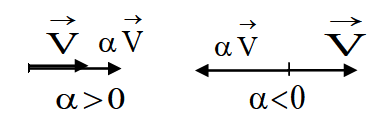
Let be three points A, B and C we have

**Special case**: If the three points A, B and C are aligned on an axis, then we obtain the Chasles relation for algebraic measures

**- Multiplying of a vector by a scalar**

Multiplying a vector by a scalar (a real number) changes its magnitude but not its direction.

The product of a vector by a scalar is a vector, denoted

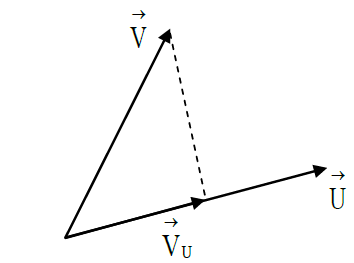


**1.2.2.2. Dot Product (Scalar Product):** The dot product of two vectors A and B is a scalar quantity defined as = |U| |V| cos(θ), where |U| and |V| are the magnitudes of the vectors, and θ is the angle between them. The dot product measures how much two vectors are aligned.

The scalar product is therefore positive for an acute angle and negative for an obtuse angle.

**Geometric shape of the scalar product**

By definition the scalar product is the algebraic projection of on

****

**Analytical form of the scalar product**

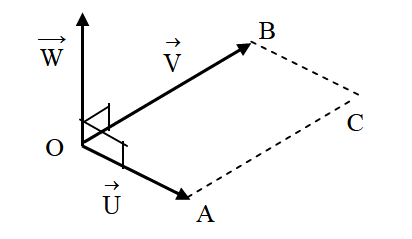
Let be two vectors

in the orthonormal basis, the scalar product of these two vectors is defined by the relation

with

**1.2.2.3. Cross Product (Vector Product):** The cross product of two vectors A and B is another vector, denoted by . It is defined as *= |U| |V|* sin(θ) , where *|U| and |V|* are the magnitudes of the vectors, θ is the angle between them, and n is a unit vector perpendicular to the plane formed by A and B, following the right-hand rule. The cross product yields a vector that is perpendicular to both A and B.

The **vector product** of two vectors denoted,  is the **vector** defined by its magnitude



**-**steering such that

-of path such that the trihedron is direct

**-**module

measures the area of the OABC parallelogram constructed on the representatives , of the vectors

and

**Analytical form**

Let be two vectors in the orthonormal basis, the scalar product of these two vectors is the **vector** defined by the relation

Therefore

**Properties**

-For two vectors to be parallel:

-For two vectors to be perpendicular:

**1.2.2.4. Applications of the vector product in geometry**

Knowing that the area of the parallelogram (ABCD) is given by

The area of the triangle (ABC) is therefore equal to

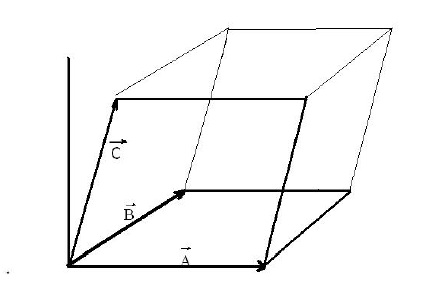
- Cartesian equation of a line (D) passing through two points A and B of a plane (x O y)

If a point then (M belongs to the set D)

**1.2.2.5. Mixed product of three vectors**

Let be three vectors the mixed product of these three vectors taken in this order is:

**Geometric interpretation of the mixed product**

The mixed product represents the volume of the parallelepiped formed by these three vectors. 

**Applications**

Let be three vectors with

1. Calculate

**Solution**

**1.3. First-order linear differential operators**

The gradient, divergence and curl are the three main linear differential operators of the first order.

**1.3.1. Gradient in the Cartesian coordinate system**

The gradient of a scalar field (a function that assigns a scalar value to each point in space) is a vector field that points in the direction of the steepest increase of the scalar field at a given point.

The gradient of a function f thus expressed represents the variation of a physical quantity in space. It is given in Cartesian coordinates by:

(Partial derivative with respect to x, y, z times the unit vector i, j, k)

Note that the gradient applies to a scalar, and its result is a vector.

**1.3.2. Divergence in the Cartesian coordinate system**

The divergence of a vector field measures how much the vectors at a point in the field spread out or converge. The divergence operator is a linear differential operator with prime partial derivatives. It transforms a vector field into a scalar field defined by:

The divergence of vector A is equal to the dot product of the gradient operator ( and vector , which is equal to the sum of the partial derivatives of the vector components and with respect to x, y, and z, respectively.

**1.3.3. Curl in the Cartesian coordinate system**

The curl of a vector field measures its rotation at a given point. It is another vector field. The curl operator is a differential operator with partial derivatives applied to a vector field

, matches another noted field:

**1.3.4. The Laplacian**

The Laplacian is defined as the divergent of the gradient of a scalar field

- The Laplacian of an algebraic function is given by the following relation:

The Laplacian of a vector function is given by the following relation:

is equal to the second partial derivative of the function f with respect to x squared, plus the second partial derivative of the function f with respect to y squared, plus the second partial derivative of the function f with respect to z squared."

This expression describes the Laplacian operator, which quantifies how a scalar field varies in three-dimensional space.

Top of Form

**Applications**

1-Let and be two vector and scalar fields respectively

and

2-Let be the vector field

Show that

**Solution**

3- Let be the two vectors and

find values for *α* and *β* that make vector parallel to vector , and then calculate the unit vectors for both of these vectors.

Top of Form

**Solution**

The two vectors are parallel so

Therefore

**Chapter 2. Kinematics**

**2.1. Introduction**

Kinematics is the science that describes the movement of a body, that is, the apparent change in its position over time, without being interested in the causes. Most of the bodies studied by physicists are in motion that appears at all scales of the universe, from particles such as electrons, protons and neutrons, to galaxies. It is essential to properly define the movement in order to understand the phenomena observed.

A body can have a translational movement (the movement of a car on a road), rotation (that of the earth on itself), vibration (small oscillations of a mass-spring system) or a combination of these movements.

**2.2. Reference system**

Rest and movement are two relative notions. Indeed, a tree is fixed by a stationary observer A while it is seen moving backwards by a driver B of a car driving nearby. In physics, the study of motion is carried out by replacing the observer with a coordinate system called a reference frame or reference system (fixed (A) or mobile (B)).

Rest and motion are expressed with respect to a reference frame which is an orthonormed reference frame.

R (O, x, y, z) in which is located the position M (x, y, z) of a body. The body is at rest relative to this reference frame if its coordinates are constant over time. However, if at least one of them varies the body is in motion with respect to R.

To express the notions of rest and motion with respect to a reference frame, consider an orthonormal reference frame R (O, x, y, z) in which is located the position M (x, y, z) of a field. The body is at rest relative to this reference frame if its coordinates are constant over time. However, if at least one of them varies the body is in motion with respect to R

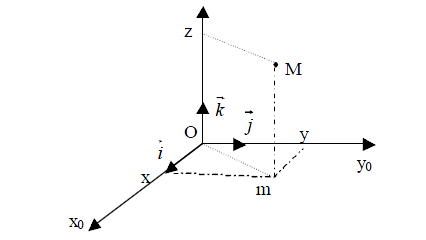
**2.3. Displacement vector**

A material point moving along a distance AB. The displacement vector represents is without reference to the origin of the reference frame.

**2.4. Position vector**

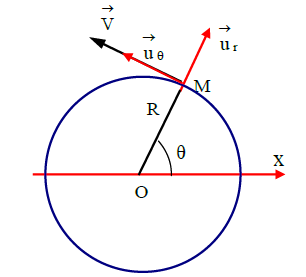
The position vector is a vector whose starting point is the origin of the reference frame and its arrival point is the material point M. It can be represented in different coordinate systems by the coordinates of the point M in each coordinate system.

**2.4.1. In Cartesian coordinates**



Unit vectors do not change directions or modulus.

**2.4.2. In polar coordinates**



We can express ourselves differently

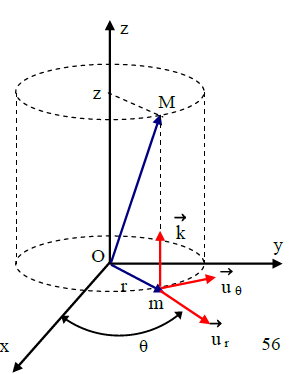
and

In other words

With

Unit vectors are time-dependent. They do not change modules but change directions.

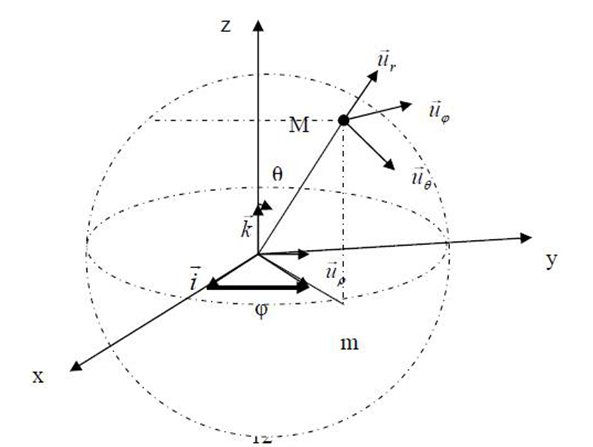
**2.4.3. In cylindrical coordinates**



and and

In other words

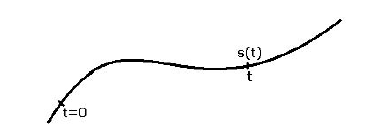
**2.4.5. In spherical coordinates**

****

These unit vectors can be expressed using matrix:

Vectors change directions but not modulus.

**2.4.6. In intrinsic coordinates**



We define the curvilinear abscissa, denoted s(t), the distance traveled on the curve with respect to a reference point (the position of the center of inertia of the mobile at t = 0s.

For a small displacement from  *M (x, y, z, t) to M'(x + dx, y + dy, z + dz, t + dt),* the curvilinear abscissa is comparable to a segment, hence

With

Therefore

Where from

The integral of the square root of the sum of the squares of the time derivatives of x, y, and z with respect to time, integrated with respect to time

**2.5. Law of movement**

The observation of a change in the position or orientation of a body is called a movement. The description of this movement is done in relation to a reference frame.

The nature of the movement of a material point is determined by the variation of its speed as a function of time.

**2.5.1. The trajectory**

It is the geometric place of the successive positions occupied by the material point over time and in relation to the chosen reference system.

**2.5.2. The speed of a material point**

Velocity measures the relationship of evolution to time. It is a vector quantity that measures the ratio of distance traveled to time. So, the velocity of a mobile is the physical quantity that expresses the ratio of the distance traveled x by the time t taken to travel it. It is expressed in meters/second (m/s) in the system (SI).

**Example**

Let be a material point moving according to the following coordinates. Find the equation of its trajectory.

and

**Solution**

The value of t is replaced in the equation of x(t). What gives

represents the equation of the trajectory of the material point.

**a-Average velocity**

The average velocity, represented as vmoy, is equal to the change in position (∆x) divided by the change in time (∆t). The ratio between the distance travelled and the time taken to cover this distance represents the average velocity:

**b-Instantaneous velocity**

The instantaneous velocity is the average velocity calculated over an infinitely small time interval. Its expression is given by:

the derivative of the position (x) with respect to time (t) and is the limit of the average velocity (v\_moy) as the time interval (∆t) approaches zero

The instantaneous speed can be determined by different methods.

**- Analytical method**

If the coordinates of a mobile are given by the law of spaces, x(t), its velocity at time t is equal to the value of the derivative with respect to the time of its hourly equation.

**Example**

(x (t) in metres and t in seconds).

Find the instantaneous velocity at t = 3s

We have the previous formula

We derive x(t) with respect to t and we find

**-Slope calculation method**

The instantaneous velocity of a mobile at a time t is equal to the slope of the tangent to the curve, representing its space diagram, at the abscissa point t. It is equal to the average velocity calculated in a sufficiently small time interval between moments

**2.5.2. Acceleration of a material point**

The acceleration therefore corresponds to the variation of the velocity vector either in modulus or direction or both.

**a-Average acceleration**

The average acceleration of a mobile expresses the variation in velocity, denoted , during a time interval

**b-Instantaneous acceleration**

The instantaneous acceleration of a mobile is equal to its average acceleration calculated over an infinitely small-time interval. Its formula is

The instantaneous acceleration can be determined by different methods:

**-Analytical method**

If the coordinates of a mobile are given by the law of spaces, x(t) or by the law of velocities, its acceleration at time t is equal to the value of the second derivative with respect to time of its hourly equation, or the first derivative of its velocity equation.

**Example**

We use the same example. Let (x(t) in meters and t in seconds).

Find instantaneous acceleration at t = 3s

**-Slope method**

The acceleration of a mobile M, at a time t, is equal to the physical slope of the tangent to the curve, representing its velocity diagram, at the abscissa point t.

**Remarks**

-By knowing the law of accelerations and the initial conditions, we can determine the velocities from the expression

By knowing the law of velocities and the initial conditions, we can determine the spaces from the expression.

**2.5.3. Diagrams of spaces, velocities and accelerations**

These diagrams represent the graphs of x(t), v(t) and a(t) respectively as a function of time.

**Recapitulation**

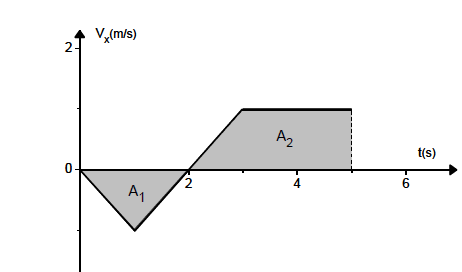
If the acceleration is constant, we have

Therefore

and

**Example**

Consider the following velocity diagram. Determine x(2s) and x(5s) knowing that x(0s) = 1m



The formula is used

Therefore

Therefore

**Remarks**

- Since the areas are algebraic, any part below the axis (Ot) is negative and the part above it is positive.

- To calculate the distance travelled between 0s and 5s, the absolute values of the areas considered are summed

**2.6. Nature of the movement of a mobile**

The nature of the movement of a material point is determined by the variation of its velocity as a function of time. The movement is said to be accelerated or delayed when the speed increases or decreases with time. The movement is said to be uniform if the velocity is constant in modulus and direction**.**

**Example of movement**

A material point M is in accelerated rectilinear motion between two points in space A and B. At the moment , it has a velocity and at its velocity is equal to . Assuming that its acceleration is constant, determine its law of velocities and spaces.

**Solution**

**-Law of speeds**

First we determine the value of the acceleration

Therefore

**Remarks**

The nature of the movement is given by the product

If the product , the movement is said to be uniformly accelerated. So, "uniformly accelerated" means that an object is moving in a way that its velocity is changing at a constant rate, specifically, it's increasing at a constant rate

If the product , the movement is said to be uniformly rectilinear decelerate. So, "uniformly rectilinear decelerate" means that an object is moving in a straight line and slowing down at a constant rate.

If the product , the movement is uniform rectilinear or at rest.

**2.7. Velocity and acceleration in coordinate systems**

**2.7.1. Cartesian coordinates**

We have defined the position vector in the Cartesian coordinate system as follows

The instantaneous velocity vector is defined by:

The instantaneous acceleration vector is defined by:

**2.7.2. Cylindrical coordinates**

and and

The instantaneous velocity vector

So

and

Where from

The instantaneous acceleration vectors

Is the radial acceleration and Is the anti-radial acceleration.

**2.7.3. Spherical coordinates**

**The velocity vector**

Therefore

Where from

**The vector of acceleration**

We have

Knowing that

So =

After term-to-term coparaison

With radial acceleration , anti-radial acceleration and axial acceleration

**2.7.4. In intrinsic coordinates**

We have

**The velocity vector**

With unit vector tangent to the curve and moving with the material point.

**The vector of acceleration**

With and is tangential acceleration and is normal acceleration

**Applications: Movement of the material point in the different coordinate systems.**

**1-Intrinsic coordinates (Frenet coordinate system)**

The plane is related to an orthonorm coordinate system of origin O and basis (). The x and *y coordinates* of a moving point *M* in the plane ( ) vary with time according to the law:

and

1/ Determine the nature of the trajectory.

2/ Determine the components of the velocity vector

3/ Determine the expression of the velocity as well as that of the curvilinear abscissa S of the point M at time t respecting the following initial conditions .

4/ Determine the normal and tangential components of the acceleration in a Frenet reference frame,

5/ deduce the radius of curvature of the trajectory.

6/ The trajectory remains the same, but now the point *M* undergoes an angular acceleration

.

At what moment will the point *M* reach a speed of , knowing that it has left rest. How far did he travel?

**Solution**

1-The trajectory

The trajectory is a circle of radius R = 2 and of origin (0, 0)

2- The components of the velocity vector

and from where

3-

and therefore from where

4- Tangential and normal acceleration

and from where

5-The radius of curvature

We have

6-Angular velocity

We have

We can deduce the linear velocity

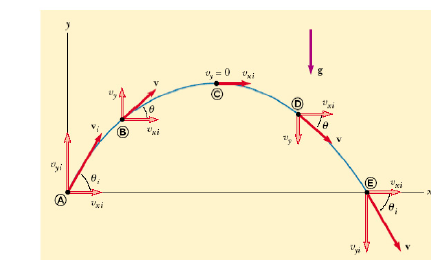
The speed reaches the value at t = ?

By integration of angular velocity we find

We find

**2-Movement of a projectile**

To study the motion of the projectile, we take as origin spaces x = 0 and y = 0 to t = 0. The acceleration following Ox is zero (ax =  0) and following Oy we have (ay = -g). The mobile is projected with an initial velocity at an angle



1-Determine the laws of speeds.

2-Determine the time equations of the movement.

3-Deduct the equation from the trajectory, the range of the projectile and the maximum height reached.

**Solution**

The laws of speeds

We apply

No acceleration on the axis (ox)

The acceleration represents –g on the axis (oy)

**Time equations**

We apply

Trajectory equation

We have

That we replace in the value of y

Therefore

is the trajectory equation

The range P of the projectile represents the value of x which corresponds to the meeting of the mobile with the ground. This value of x corresponds to the root of the polynomial

. P is the range of the projectile

The maximum height reached when the speed on the axis (oy) cancels out.

One

We replace the value of x in the value of y t we will have

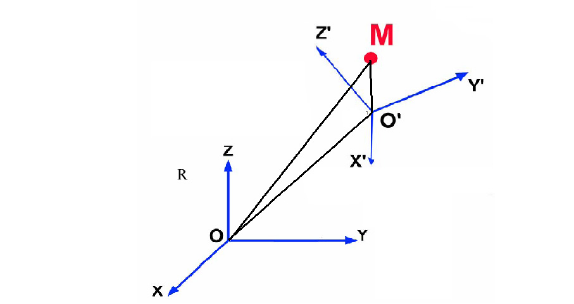
**2.8. Relative movement**

The motion is absolute if it is studied with respect to a fixed reference frame, but the motion is relative if it is studied with respect to a movable reference frame.

Movement of a material point M observed in two Galilean frames named with

,

We have



**Determination of the motion of the point M in the reference frame R**.

In R the point M has coordinates (x, y, z.)

Position vector of M in R:

The velocity vector of M in R

The vector of the acceleration of m in R

**Determination of the motion of the point M in the reference frame R'**

In R' the point M has coordinates x', y', z'.

Position vector of M in R'

Velocity Vector

The Acceleration Vector

**Determination of the motion of the point O' origin of R' in the reference frame R**

In R the point O has coordinates x0, y0, z0.

Position vector of 0' in R

The Velocity Vector

Acceleration vector

**Any motion of R' with respect to R**

Relationship between velocity vectors

is called absolute speed

is called relative velocity

Is called the training speed of R' in R.

Represents the velocity of M in R'

Therefore

**Relation between the acceleration vectors: The acceleration of M in the reference frame R is**

is called the drive acceleration of R' in R.

is called the acceleration of M in R'.

is called the Coriolis acceleration.

**Applications**

1. The coordinate system R' is in translation with respect to R when the unit vectors of the coordinate system R' do not change over time and they keep the same direction and direction as the coordinate system R:

Therefore

Where from

And

1. It is assumed that the coordinate system R' is rotating with respect to the z-axis with an angular velocity and O≡O′ is considered.

Any vector rotating with respect to a perpendicular axis, its time derivative is the vector product of its angular velocity and the rotating vector

;  ;

The training speed becomes

**For training acceleration**

We have

therefore

All the same and

Hence the drive acceleration takes the following formula

Coriolis acceleration

.

Chapter 3. Hardware Point Dynamics

**3.1. General**

**3.1.1. Introduction**

In the previous chapter, we focused on movements without worrying about the agents who cause them. In this part, we will discuss the dynamics that address the causes of movement**.** It makes it possible to determine the causes of a known movement and to predict the movement for given causes. In fact, we will establish the relations between the movement and its cause.

**3.1.2. Concept of Mass**

The mass of inertia is an intrinsic characteristic of each body. Physically it is difficult to differentiate between the heavy mass and the mass of inertia. They are always considered equivalent. The equivalence between the inertial mass and the heavy mass is not obvious but it is known from experience.

The mass of a body can be considered as a set of material points. If this set of points is isolated, it can be represented by a point that is either at rest or in rectilinear motion with respect to an inertia coordinate system. It is called the center of inertia.

There is no experiment that has succeeded in awarding the center of inertia of the center of mass or the center of gravity. They are still considered confused despite not having the same physical origin.

**3.1.2. Concept of force**

It is known that movement is the result of the interaction between the material point and its environment. This interaction, called force, is characterized by the properties of the material point (mass, charge, dipole moment...) and by the nature of its environment. Force is a quantity that translates the interactions between objects. It is a cause capable of producing the movement of a body or modifying it, or of generating its deformation. The forces can be classified as follows:

**a. Contact forces**

They reflect interactions between bodies in physical contact. They include:

- **frictional forces** : they appear when two bodies in contact are in relative motion, one relative to the other. They always oppose the movement of the body under consideration.

- **tension forces**  : these are forces that pull on an element of a body such as, for example, the tension exerted by a wire or by a spring

**b. Ranged forces**

They manifest via vector fields even if there is no physical contact between the two interacting bodies. They include:

-Gravitational forces : these are forces of attraction that are exerted between bodies and are due to their masses. Example The weight of a body and the forces exchanged by celestial bodies are essentially gravitational forces.

- **electric forces** : they are exerted between two objects carrying electric charges. They can be attractive or repellent.

- **magnetic forces** : they are exerted between magnets, between magnets and certain materials (in particular iron) or between two conductors traversed by an electric current. They can be attractive or repellent.

**3.1.3. The force vector**

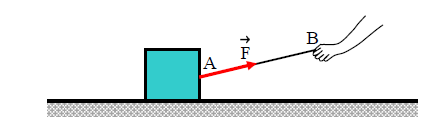
Any force can be represented by a vector that is characterized by the following four properties:

The direction: straight line according to which the action is exercised (that of the thread of the Figure below);

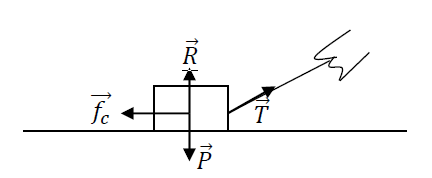
The meaning: the direction in which the action is exercised (from A to B);

The point of application: point where the action is exerted on the body (point A);

The module: the intensity of the force to which a suitable unit is associated.



**Example** : A body slides over a horizontal surface by a wire.



The forces exerted on this body are: weight force, wire tension, reaction force and frictional force

**Remark**

The forces are additive, that is, if N forces act simultaneously on a body, the motion of the latter is the same as in the case where it undergoes the action of a single force equal to the vector sum of the N forces. This sum is called **the resultant** of the N forces.

**3.2. Fundamental interactions**

Despite their great diversity, the forces encountered in nature are the manifestations of the four fundamental interactions:

**3.2.1. Gravitational interaction:** it is manifested by a force of attraction between all particles. This force appears in most of the phenomena described by astronomy and geology (the movement of the stars, the rise of the tides; the bodies attracted by the Earth in its vicinity, the non-disintegration of the Earth ...).

**3.2.2. Electromagnetic interaction:** it manifests itself between electric charges in all phenomena involving electricity and/or magnetism.

**3.2.3. The strong interaction:** this is the interaction between the nucleons that are the constituents of the nucleus of an atom. It allows particles composed of quarks, such as protons and neutrons, not to disintegrate. It is exercised at a very short distance and is responsible for the cohesion of the nucleus.

**3.2.4. The weak interaction:** it applies to all particles of matter (quarks, electrons, neutrinos, etc.). In particular, neutrinos, which are electrically neutral and are not quarks, are sensitive only to weak and gravitational interactions. The weak interaction is manifested in certain types of nuclear reactions such as radioactivity.

**3.3. Moment of force**

The moment of a force with respect to an axis of rotation is called the product of the norm F of the force and its lever arm a. It represents the ability to rotate a mechanical system around this point.

The moment of a force with respect to point A is defined by the formula:

With B is any point in the line of action of the force

**3.4. Newton's principle of inertia**

Any body (material point) not subject to force (free body) is either at rest or in rectilinear motion at constant speed.

**3.5. Mechanically isolated body**

In the absence of external forces, the body is said to be free or mechanically isolated. A system is a set of bodies identified from the rest of the universe. A system is isolated when the result of external forces applied to it is zero.

**3.6. Inertia or Galilean reference frame**

An inertial (or Galilean) reference frame is a reference frame in which a body that is not subjected to any force or that undergoes forces whose resultant is zero, will be at rest or in uniform rectilinear motion**.**

To define it, consider the kinematic state of a free particle, with respect to a moving frame of reference. Its relative acceleration is given by:

Be, respectively, the absolute, drive and Coriolis accelerations.

If the coordinate system is in uniform rectilinear motion.

Rectilinear character implies and

Uniformity results in either

The particle being isolated

Thus, a free particle does not undergo acceleration in a fixed frame or in uniform rectilinear motion. The latter is therefore Galilean.

In the vast majority of common experiments, a land-bound frame of reference can be considered an inertial frame of reference.

**3.7. Momentum**

In addition to the kinematic characteristics of a movement, it can be influenced by the mass of the mobile. The concept of momentum thus provides a quantitative distinction between the motions of two particles of the same velocity but different masses. It is the quantity that combines a kinematic property of motion, velocity, and mass**.**

The momentum of a particle is defined as the product of its mass by its velocity vector.

For a system consisting of N particles, its total momentum is defined as the vector sum of the momentum of each of the particles.

Thus, momentum is a vector having the same direction and direction as velocity and has as its unit in SI the (kg m/s).

**3.8. Conservation of momentum**

By definition, an isolated body is endowed with inertia and, therefore, moves with a constant speed, which implies that it retains its momentum.

**3.9. Newton's laws**

Newton's three laws are the basis of classical mechanics. These laws have been postulated without demonstration but they are in such agreement with experience that their validity could not be doubted.

1. **The first law:** This is the principle of inertia

If an object isolated and at rest or in uniform rectilinear motion then

**b. The second law or the fundamental relationship of dynamics**

In Newtonian mechanics, the time derivative of the vector of the momentum of a material point is equal to the sum of the forces exerting on it. Mathematically, this translates into the relationship:

This is the fundamental relationship of dynamics with:

The result of the forces applied to the body.

The amount of movement of the body in motion.

If the mass is constant we will have

This relationship associates the kinetic term which is acceleration and the dynamic term which is the forces exerted

So if we know the forces (the resultant of the forces) we can determine the nature of the movement of a given material point.

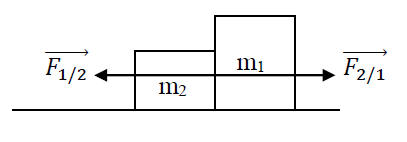
If we project on the axes of an orthonormed reference frame we will have

Therefore

1. **The Third Law (Principle of Action and Reaction)**

When two bodies are interacting, they exert opposite forces on each other in direction but equal in intensity and direction.

**Example** : Two blocks affect each other.



The force exerted on one body is called action and the force exerted on the other body is called reaction.

So every force is associated with a reaction.

The forces are of the same nature. It should not be confused with the force of weight and the force of reaction (these two forces are not of the same nature).

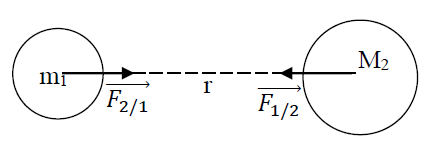
**Properties**

-The two forces act along the right, called the line of action, joining the two bodies.

-Action is the force exerted by one of the two bodies and reaction is the force exerted by the other.

**d. Fourth Law or Universal Gravitation**

The law of gravitational force, exerted between two bodies assimilated to material points, is proportional to their masses and is inversely proportional to the square of the distance between them. It is called: law of universal gravitation corresponds to Newton's 4th law.



and with

In other words , with

is the universal gravitational constant

**Application**

Calculate the gravitational force exerted by the Earth on an object of mass m = 20 kg, the mass of the Earth = , the radius of the Earth and Near the surface of the Earth.

**Solution**

From this law derives Kepler's law and which is in fact only Newton's third law, its statement is: with the dimension of k is

*The square of the period of revolution of a planet in the solar system is proportional to the cube of the mean radius of its orbit* **Demonstration**

K a constant determined by applying Newton's second law:

with

Mass of the Earth

Mass of the object and r is the semi-major axis of the elliptical path

Where from  with

**Application**

Calculate the period of the object in question

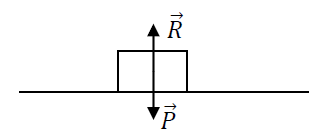
and

First we calculate the constant k

And if we have two planets we will have

**3.10. Contact force**

Let be a block on a horizontal plane



The reaction force is the action of the support on which the system rests and which prevents it from sinking downwards under the action of its weight.

If there is no movement

**3.11. Frictional force**

When a body is moving on a surface or in a viscous medium such as air and water, resistance to this movement is observed on the part of the surrounding environment. We call such resistance frictional force.

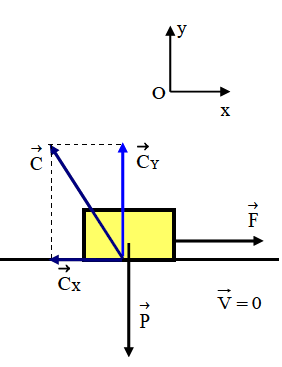
When a body moves on the surface of a solid medium or in a viscous medium such as air or water, it experiences a frictional force from the medium. These frictional forces depend on the speed of movement as can be simply constant. This part studies the frictional forces exerted by a surface on a solid body. These frictional forces are very important in our daily lives. They allow us to walk or run and they are necessary for the movement of wheeled vehicles. Although details of friction are quite complex at the atomic level, this force ultimately involves an electrostatic interaction between atoms or molecules.

There are two frictional forces between solid bodies, namely static rubbing forces , and slip friction forces or (dynamic).

If the body undergoing friction is at rest, this is the static frictional force. If the body moves it is the dynamic frictional force

**Static friction**

**First case: adhesion**



The body is in equilibrium, being vertical and horizontal, the application of Newton's second law then highlights a contact force such that

Either

The geometric projection of this equation onto the axis (OX) gives:

is an adhesion force that opposes and the eventual movement of the object to the right. Moreover this force is tangent to the contact surface, it is by definition a frictional force. As the body does not move it is said that there is adhesion and is a static frictional force.

**Second case: the disruption of the balance**

If the force becomes large enough, there is a limit value from which the balance is broken and the object begins to slide on the plane. This value is associated with a limit adhesion that allows us to define the static coefficient of friction

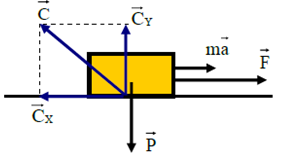
The static coefficient of friction is characterized by the following properties:

-This dimensionless cocoefficient expresses the ratio of the modulus of the frictional limiting force and that of the pressure force

-It depends on the nature of the surfaces in contact and does not depend on the area of its surfaces

-It is determined experimentally

**Dynamic friction**

****

It is the one that opposes a resistance when the object placed on the plane is already in slide.

The fundamental law of dynamics is written

either

has a tangential component that opposes the movement of the body which, by definition, is a dynamic frictional force. As in the static case, the coefficient of dynamic friction or slip is introduced.

The dynamic coefficient of friction is characterized by the following properties:

-Its value is determined experimentally

-

- is substantially independent of speed.

- depends only on the nature of the surfaces in contact

The coefficients of friction are defined:

-Static valid just at the break of equilibrium

-Dynamics valid when the body is in motion

**Exercise 1**

An object of mass m = 1kg, placed on a horizontal ground, is fired with an inclined force of one angle. Its contact with the ground is characterized by static and dynamic friction coefficients

Calculate the intensity of the force in the following three cases

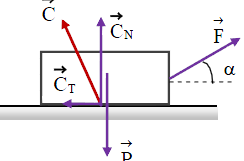
1. At the time of the disruption of equilibrium

2. The body is in uniform motion

3. its motion has an acceleration of

**Solution**

First we represent the forces acting on the mass



The application of Newton's second law gives:

Projection on the two axes of Cartesian coordinates

We can write

We could write again

Where from

1st case: equilibrium break (a = 0 m / s2 and  ) and therefore *F = 4, 48N*

2nd case: uniform movement (a = 0 m / s2 and ) and therefore *F = 2.07N*

3rd case: uniformly accelerated movement (a = 1 m / s2 and ) and therefore *F = 3.10N*

**Exercise 2**

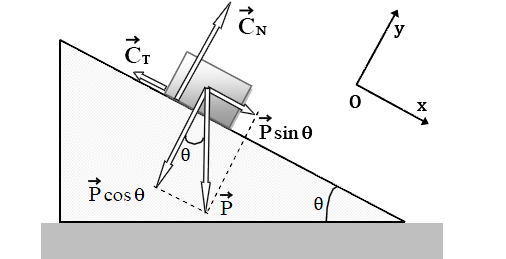
A block of mass m = 1kg is placed on a rough surface inclined at an angle to the horizontal. By gradually increasing the value of the angle we had the following observations:

-The block started to slide when the angle reached the value

- Its acceleration was for

From this deduce the values of the static and dynamic friction coefficients.

**Solution**



The application of Newton's law gives:

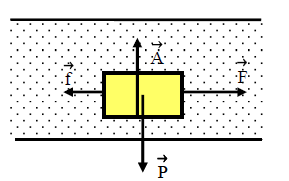
Let us project this equation on the two axes of Cartesian coordinates

1st case: equilibrium break (a = 0 m / s2 and ) and therefore

3rd case: uniformly accelerated movement (a = 3.2 m / s2 and ) and therefore

**Viscous friction**

Consider the supposedly horizontal motion of a body immersed in a fluid and subjected to a horizontal and constant force. The fluid exerts on the body a frictional force of the shape



Other forces applied to the mobile are weight, Archimedes' thrust and tractive force.

As the motion is maintained on the horizontal plane, this means that and compensate each other and by application of Newton's second law we have:

Algebraically we will have

So we will have

Taking the exponential

B is an integration constant that can be obtained, for example, by using initial conditions. If the muzzle velocity is zero then

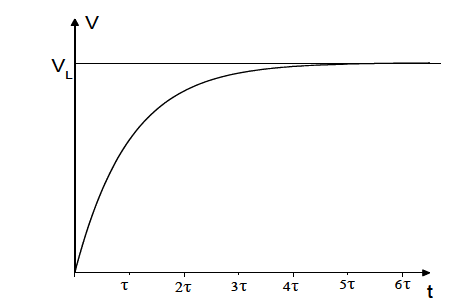
consequently

If we pose and

We find

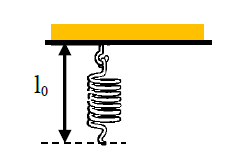
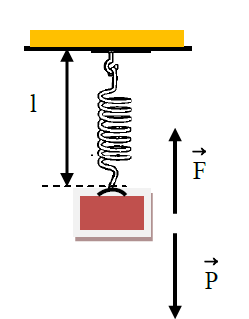
Thus, the speed increases from its initial value to an asymptotic value VL called limit speed.

We also observe an interesting behavior: a heavy body takes longer to reach the speed limit than a light mobile body.



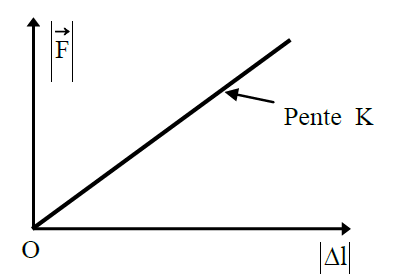
**Elastic forces**

Consider the mass-spring system. At the lower end of the spring, of length to empty l0 and suspended from the ceiling, is hung a mass which makes it undergo an elongation to achieve a balance.

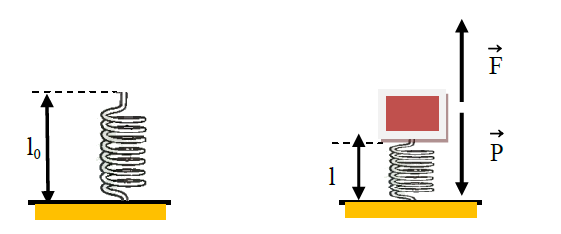
 

According to Newton's second law, the weight must be compensated by a force exerted by the spring on the mass in the opposite direction to the elongation:

By varying the mass and measuring the corresponding elongation, one can plot the graph of the variation of the modulus of the force as a function of the elongation. We obtain a line segment of slope K.



All the same for a compression.



We deduce the elastic force

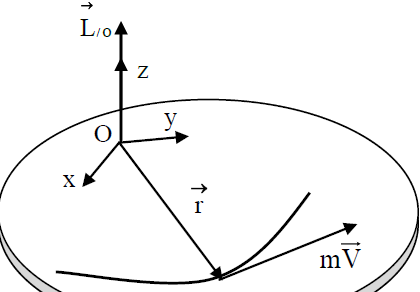
**3.12. Angular momentum of a particle**

**Definition**

Let be a particle of mass m at a point identified by the position vector and moving at speed Its angular momentum, with respect to the origin O, is defined by

being the momentum

By its definition, the angular momentum is a vector perpendicular to the plane containing the vectors and and oriented so that the trihedron is direct and its modulus is



In the case of a circular motion of radius r centered at the origin, the position vector is always perpendicular to the direction of the velocity vector and thus

Since angular momentum has the same direction and direction as the angular velocity vector, it becomes obvious that:

For any curvilinear plane motion, we introduce the radial and transverse components of velocity, into a polar coordinate system of pole O

Given that is parallel to we obtain

and in the scalar form:

**Angular momentum theorem for a particle**

The derivative of angular momentum is obtained by applying the rules of derivation of function products

We have

And so

**The angular momentum theorem for a particle**

The time-derived derivative of the angular momentum of a moving particle is equal to the moment resulting from external forces, relative to the reference point used for angular momentum.

This theorem is analogous to Newton's second law.

**Remark**

Analysis of the previous relationship shows that the derivative of angular momentum with respect to time is zero in two cases**:**

- The particle is isolated, i.e. the resultant of the external forces is zero. As a result, the angular momentum of a free particle is constant.

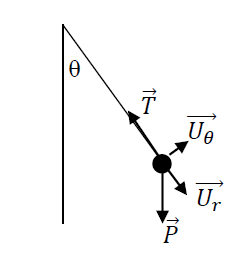
-The angular momentum with respect to the center of forces is constant if the force is central.

**Exercise**

A point mass m suspended from an inextensible wire of length l is removed from its equilibrium position. The position of the mass m is identified by the angle θ between the vertical and the direction of the wire. Establish the differential equation of motion using:

1)- The fundamental principle of dynamics (using the polar coordinate system).

2)- The angular momentum theorem.

**Solution** 

The forces applied are: wire tension and weight.

1)- By applying the fundamental principle of dynamics

Acceleration in polar coordinates of writing

In this case, we have: r = l so

By projection on and

Where we will have

1. By applying the angular momentum theorem

and

The velocity in polar coordinates is

and

This implies that

The moment of the forces is

Chapter 4: Work and Energy

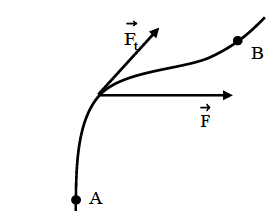
**4.1. Work of a force**

**4.1.1. Definitions**

If a particle undergoes an elementary displacement under the effect of a force , the latter performs an elementary work defined by:

Note that the work is a scalar product of the force vector and the displacement vector

If the particle is moved from point A to point B, the total work is



**Remark**

With the curvilinear coordinate and the tangential component of the force.

If the force is perpendicular to the displacement its work is zero.

The work is an algebraic quantity: if is positive the work is called motor work; otherwise it is called resistant work. The work usually depends on the path taken between A and B.

If we use the Cartesian coordinates the work takes the following formula:

Hence the work

If the force is constant we can write

This result shows that the work then depends only on the initial and final positions and not on the path followed. Weight work is a good example.

**4.1.2. Applications**

**(a)**  A particle is subjected to force.

Calculate the work performed by force when the particle moves from point (0, 0) to point (2, 0) along a straight line

**Solution**

From the data we see that the particle moves parallel to the OX axis and therefore

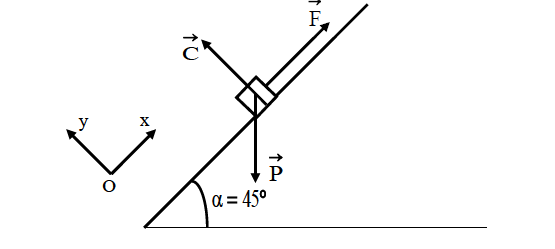
Now we can easily calculate the work done

This can easily be deduced since the force is perpendicular to the displacement vector such that

and therefore

**(b)** A body of mass is pulled with a force over a distance of 10 m, along a plane inclined 45° from the horizontal. The friction being insignificant and the movement is accelerated, acceleration.

Calculate the work of the forces applied to the body of mass m 1kg**.**

****

**Solution**

The application of Newton's second law makes it possible to write

By projection we will have

We can write

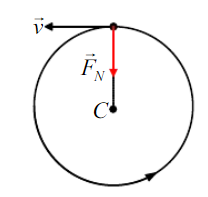
YEAR

The work of this force is

Weight work is

The work of force is null because

**(c)** A mobile moving along a circle with a velocity . This motive is subjected to a force according to the following figure. Calculate the work of this force.



**Solution**

In circular motion, the work of normal force is zero.

**4.2. Kinetic energy**

**4.2.1. Definition**

Let be a body moving under the action of a force between two points A and B. According to Newton's second law we have

**Theorem**

The work of the resultant, of all forces (conservative and nonconservative) applied at a material point M, in any reference frame R, between the initial position A and the final position B, is equal to the variation of its kinetic energy between A and B.

In other words:

Represents the relation of the kinetic energy theorem. When a body moves between two points A and B, under the action of a resultant force, the work of this force is, whatever the path followed and the nature of the forces, equal to the variation in the kinetic energy of the body.

If we use the momentum of the movement

It comes that

If this force is perpendicular to the displacement its work is zero and consequently the kinetic energy is constant.

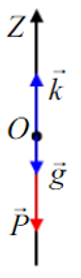
**4.3. Conservative forces or forces deriving from potential**

**Definition**

A force is said to be conservative or derived from potential if its work is independent of the path followed regardless of the movement between the starting point and the end point.

If trajectory C is closed then the work following this trajectory is zero.

**Example 1.** In a Cartisian coordinate system where Oz is the upward-facing vertical such that



Using the expression of elementary displacement in cartisene coordinates we find

We realize that the work for a displacement between two points A and B does not depend on the path followed but depends only on their altitudes z1 and z2.

If the two points are in the same plane, the work done by the weight is zero, which proves that the weight is a concervative force.

**4.4. Non-conservative forces**

A non-conservative force is a force that does not derive from potential. So the work of this kind of force depends on the path followed. Frictional forces are non-conservative forces.

**Example 2.** The force can go from point A (0, 0) to point B (2, 4) following the two paths indicated

Say if this force is conservative.

**Solution**

* Following the first path

Therefore

* Following the second path

It is deduced that the force is not conservative because the works are not equal.

**4.5. Potential energy**

**Definition**

The potential energy is a function of coordinates such that the integration between its two values taken at the start and at the end is equal to the work done to move it from its initial position to its final position.

If the force derives from a potential then

The potential energy is always related to a reference frame taken as the origin to calculate it (. The function of the potential energy is determined to a constant.

Consider the function , its differential is

By definition the potential energy differential is equal and opposite to the work differential.

**4.6. Particulate matter in the uniform Earth's gravity field**

If z is the height, defined from the surface of the earth taken as the origin of the potential energy, then the potential energy of the particle with respect to the surface of the earth is:

**4.7. Particle subjected to elastic force**

We will calculate the potential energy of a system consisting of a particle attached to a spring, suspended vertically from stiffness constant k, its empty length being l0. Its length when charged with the particle is l

We have

**Example**

Calculate the work required to lengthen a vertically suspended spring by 3 cm without any acceleration knowing that the stiffness constant is k = 50N.m-1

**Solution**

Strength

**4.8. Mechanical energy**

**Definition**

The mechanical energy of a material point at a given time is equal to the sum of kinetic energy and potential energy

**Example**

The mechanical energy of a system composed of a spring of stiffness constant k whose elongation is at time t under the action of a particle of mass m and instantaneous velocity *v* is

**4.9. Principle of conservation of mechanical energy**

In the conservative force field (or derived from a potential) mechanical energy is conserved over time.

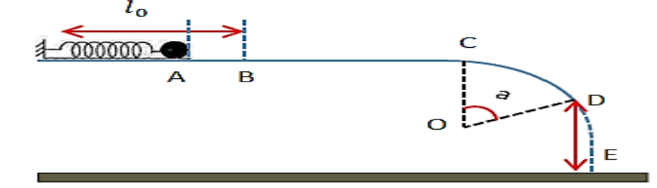
This means that the variation of mechanical energy is zero, which means that

Either a mechanically isolated system, its mechanical energy is conserved.

In the case of the presence of friction, the variation of mechanical energy is equal to the sum of the work of the frictional forces

**Exercise**

Let be the following figure where the movement of a ball of mass m = 30 g is configured. First We compress a spring of stiffness constant of k = 3N / cm and initial length l0 by this ball and then it is released without initial velocity of the position A.



1. Calculate the velocity of the ball at the moment of spring separation (point B).
2. The ball continues its movement on the rough horizontal plane to arrive at point C with a speed vc = 3.5 m / s. Then calculate the frictional force in the section BC knowing that BC = L = 50 cm.
3. The CD part of the trajectory represents an arc of a circle of origin O and radius r = 6 cm

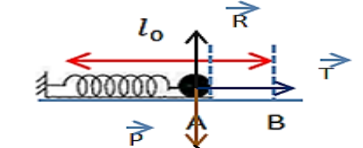
Arriving at point D, the ball leaves the trajectory with a speed v D = 6.6 m/s.

(a) Calculate angle a.

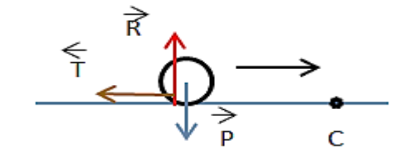
(b) Calculate the velocity of the ball when hitting the ground at point E.

**Solution**

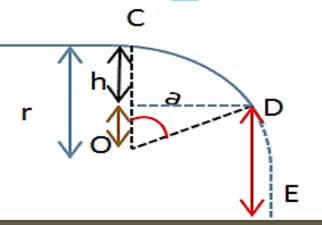
1. Energy Conservation



1. Calculation of the frictional force



1. (a) Calculation of angle a



(b) Calculation of speed at point E

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